III Semester M.Sc. Degree Examination, December 2014
(Y2K11 Scheme) (RNS)
MATHEMATICS
M 301 : Topology – II

Time : 3 Hours   Max. Marks : 80

Instructions : *i) Answer* ***any five*** *questions, choosing atleast* ***two*** *from* ***each*** *Part.*
ii) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Show that a continuous image of a compact space is compact. **4**

   b) Define a countably compact space. Show that every compact space is countably compact. Is the converse true ? Justify your answer. **7**

   c) Prove that countably compactness does not imply sequentially compact. **5**

2. a) Prove that every second axiom space is a first axiom space. Show that converse is false. **6**

   b) Prove that a metric space (X, d) is countably compact iff every countable open cover has a finite subcover. **6**

   c) Prove that a compact metric space is complete. **4**

3. a) Show that the product space $X \times Y$ is Hausdorff space iff X and Y are Hausdorff spaces. **6**

   b) Show that $X \times Y$ is locally connected iff X and Y are locally connected. **5**

   c) Prove that $X \times Y$ is completely regular iff X and Y are completely regular. **5**

4. a) Define T_0 – space. Prove that in a T_0 – space the closure of distinct points are distinct and conversely. **6**

   b) Define T_2 – space. Show that T_1 – space does not imply T_2 – space. **5**

   c) Show that a point x in a T_1 – space (X, J) is a limit point of a subset A of X if and only if every open set containing x contains infinitely many distinct points of A. **5**

PART – B

5. a) Define T_3 – space. Prove that every metric space is a T_3 – space. **4**

b) Prove that a space (X, J) is regular iff given any open set G and $x \in G$, there is an open set G* such that $x \in G^* \subseteq \overline{G^*} \subseteq G$. **6**

c) Prove that complete regularity is both topological and hereditary. **6**

6. a) Define normal space. Prove that a closed subspace of a normal space is normal. **6**

b) Prove that a compact Hausdorff space is normal. **6**

c) Is the normality property hereditary ? Justify your answer. **4**

7. a) State and prove Tietze's extension theorem. **12**

b) Prove that a normal space is regular iff it is completely regular. **4**

8. a) Prove that a space is completely normal iff every subspace is normal. **9**

b) Define paracompact space. Prove that every paracompact space is normal. **7**